TRINITY COLLEGE

Year 12 Specialist Mathematics Units 3,4
Test 1 2019

Section 1 Calculator Free
Complex Numbers, Functions

STUDENT'S NAME ____________________

DATE: Wednesday 6th March **TIME:** 50 minutes **MARKS:** 53

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, one A4 page of notes

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Given $f(x)=\sqrt{x}+2$ and $g(x)=x^2-1$

Determine the domain and range of $y=f(g(x))$

$f \circ g(x) = \sqrt{x^2-1} + 2$

$x^2-1 \geq 0$

$x^2 \geq 1$ $x = \pm 1$

$D: x \geq 1, x \leq -1 \quad x \in R$

$R: y \geq 2 \quad y \in R$

2. (7 marks)

For the expression $2z^4 - z^3 + 13z^2 - 4z + 20$

(a) show $z - 2i$ is a factor of the expression [2]

$z = 2i$ $\quad P(2i) = 2\times(2i)^4 - (2i)^3 + 13(2i)^2 - 4(2i) + 20$

$= 32 + 8i - 52 - 8i + 20$

$= 0$

$\therefore$ FACTOR

(b) state another factor of the expression [1]

$z + 2i$

(c) hence solve $2z^4 - z^3 + 13z^2 - 4z + 20 = 0$ [4]

$(z-2i)(z+2i)(az^2+bz+c) = 2z^4 - z^3 + 13z^2 - 4z + 20$

$(z^2+4)(2z^2 - z + 5) = 2z^4 - z^3 - 13z^2 - 4z + 20$

$2z^2 - z + 5 = 0$

$z = \dfrac{1 \pm \sqrt{1-40}}{4}$

$= \dfrac{1 \pm \sqrt{39}\, i}{4}$

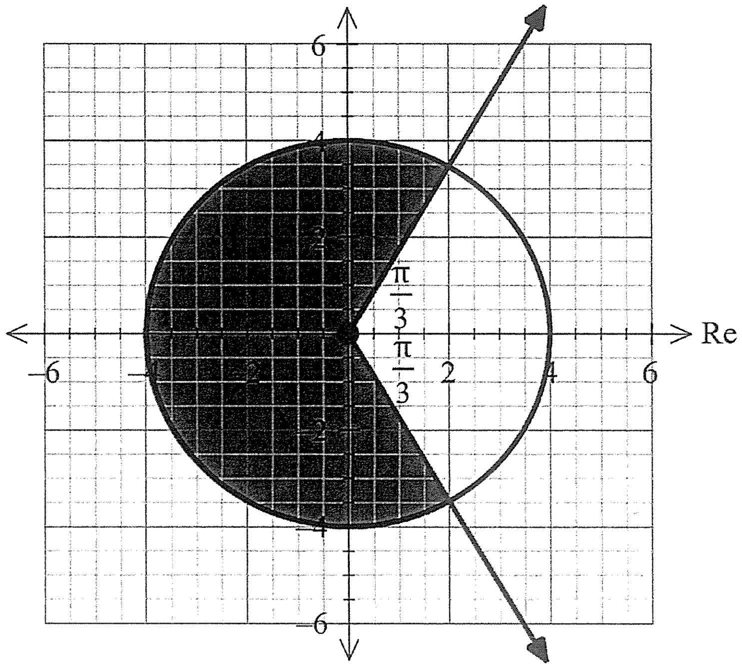
$z = \dfrac{1 \pm \sqrt{39}\, i}{4}, \quad \pm 2i$

3. (8 marks)

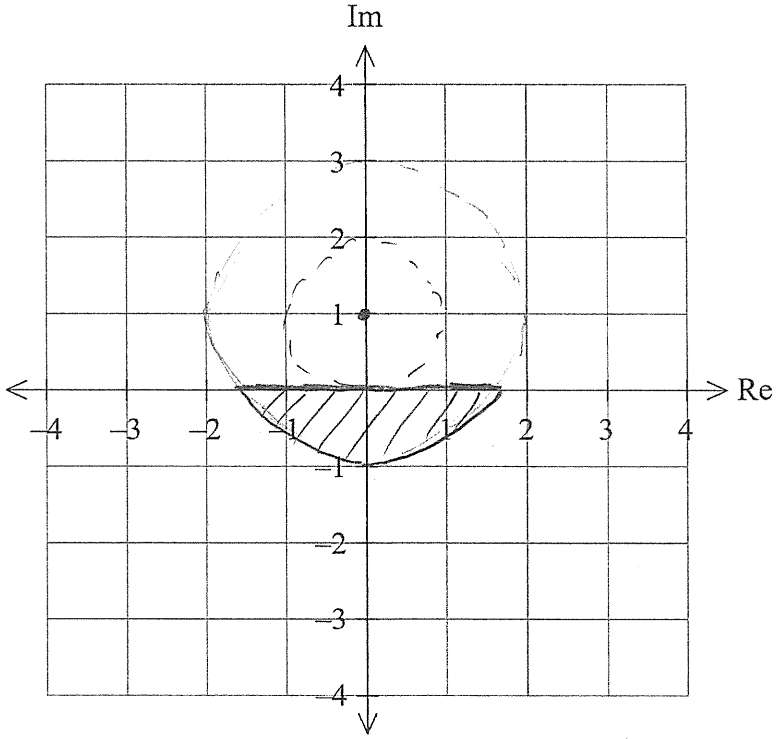
(a) Describe fully the shaded region show. [3]

$|z| \leq 4$

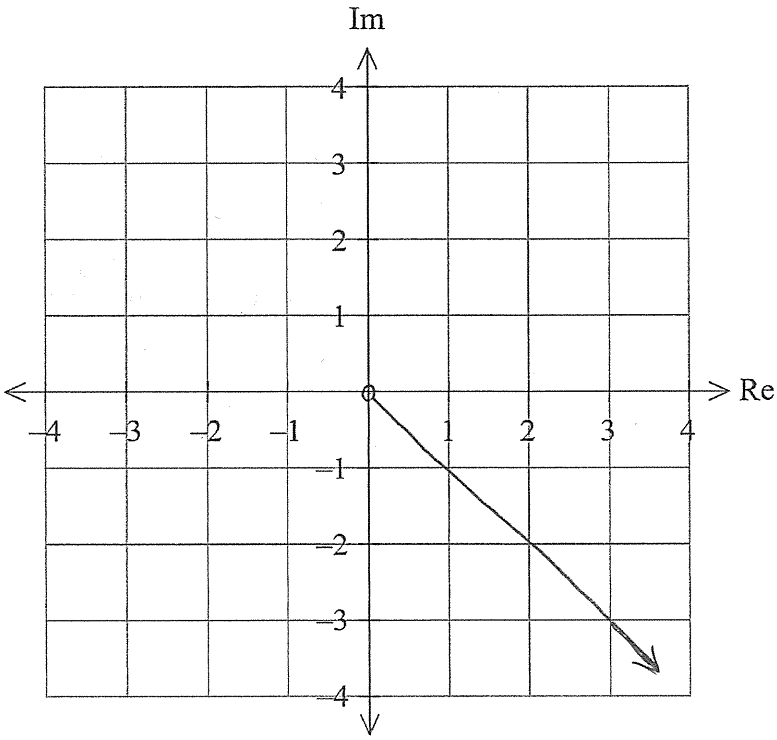
$|Arg\, z| \geqslant \frac{\pi}{3}$



(b) Sketch $1 < |z-(i)| \leq 2 \;\cap\; -\pi \leq Arg(z) \leq 0$ [3]



(c) Sketch $Arg(z)+Arg(2+i\sqrt{2})=0$ [2]



4. (6 marks)

Solve $z^5 = \frac{-i}{32}$. Answer may be given in polar form.

$$z^5 = \frac{1}{32} \operatorname{cis}\left(-\frac{\pi}{2}\right)$$

$$z = \frac{1}{2}\left[\operatorname{cis}\left(-\frac{\pi}{2}\right)\right]^{\frac{1}{5}}$$

$$z_1 = \frac{1}{2} \operatorname{cis}\left(-\frac{\pi}{10}\right)$$

$$z_2 = \frac{1}{2} \operatorname{cis}\left(\frac{-\frac{\pi}{2} + 2\pi}{5}\right)$$

$$= \frac{1}{2} \operatorname{cis} \frac{3\pi}{10}$$

$$z_3 = \frac{1}{2} \operatorname{cis}\left(\frac{-\frac{\pi}{2} + 4\pi}{5}\right)$$

$$= \frac{1}{2} \operatorname{cis} \frac{7\pi}{10}$$

$$z_4 = \frac{1}{2} \operatorname{cis}\left(\frac{-\frac{\pi}{2} + 6\pi}{5}\right)$$

$$= \frac{1}{2} \operatorname{cis} \frac{11\pi}{10}$$

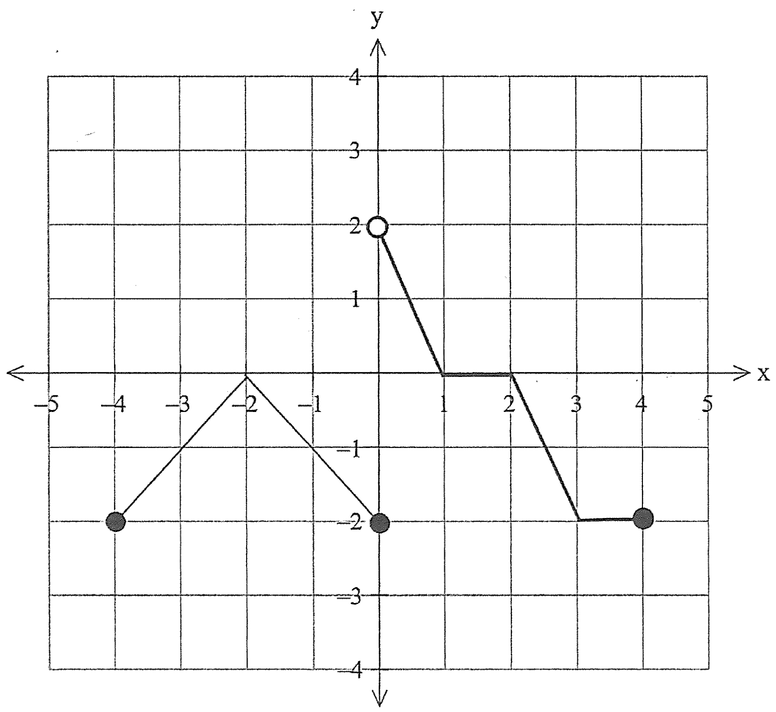
$$= \frac{1}{2} \operatorname{cis}\left(-\frac{9\pi}{10}\right)$$

$$z_5 = \frac{1}{2} \operatorname{cis}\left(\frac{-\frac{\pi}{2} + 8\pi}{5}\right)$$

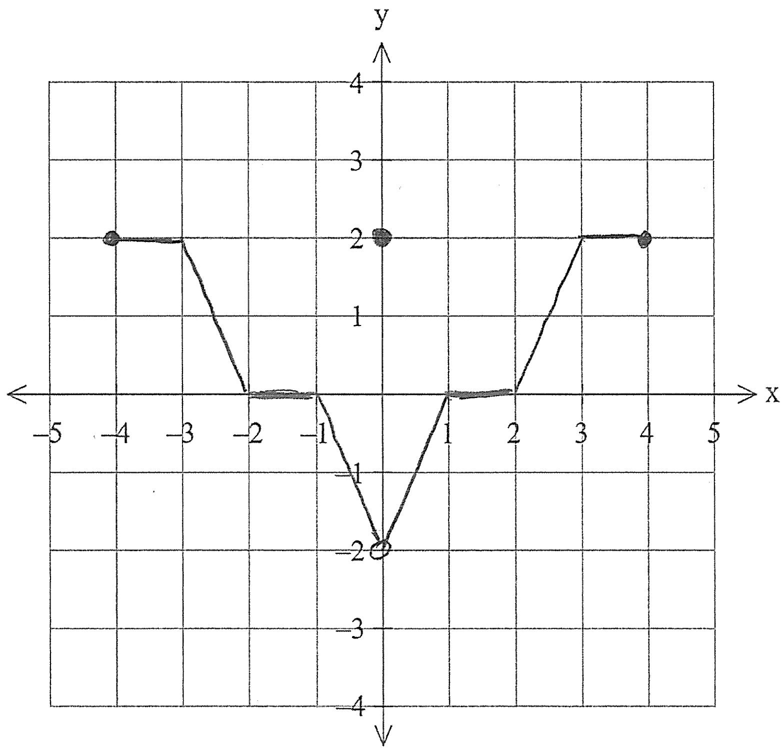
$$= \frac{1}{2} \operatorname{cis}\left(-\frac{\pi}{2}\right)$$

5. (7 marks)

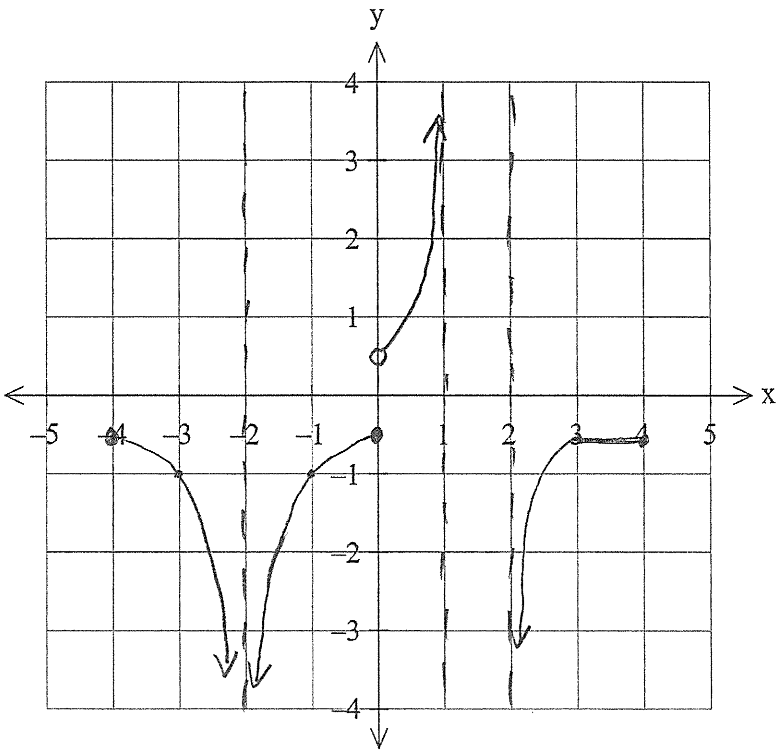
Given $y = f(x)$ as shown below



(a) sketch $y = -f|x|$ [2]

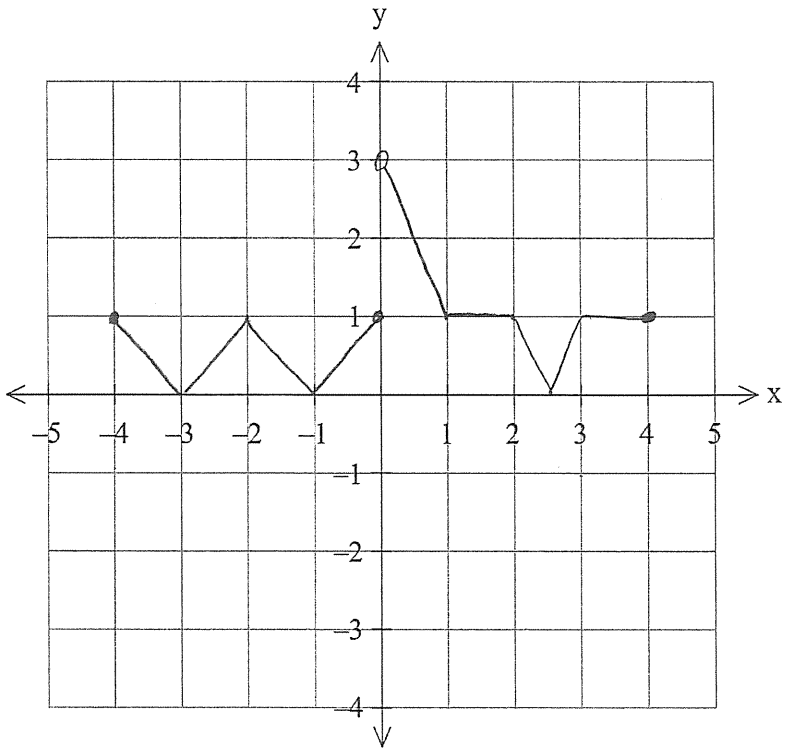


(b) sketch $y = \frac{1}{f(x)}$ [2]



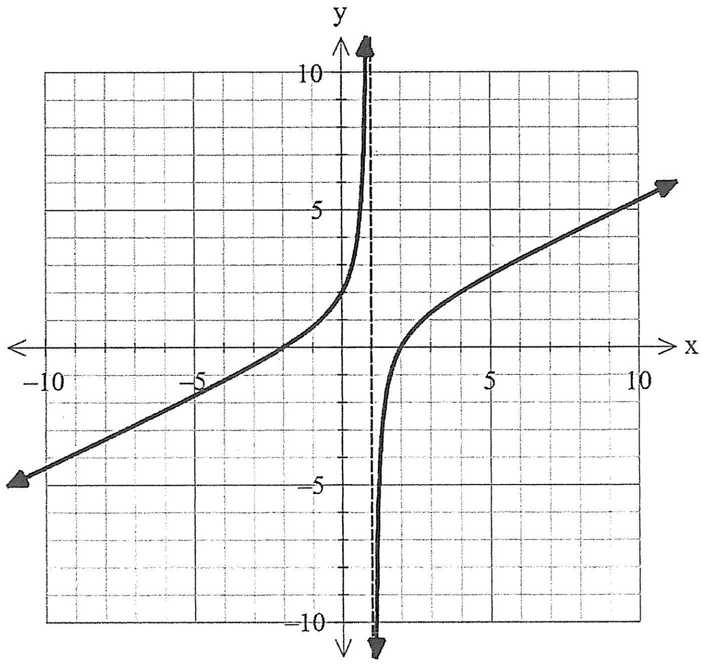
(c) solve $|f(x)+1| = 2$ [3]

$x = \frac{1}{2}$



6. (8 marks)

The graph below shows the function $f(x)=\dfrac{ax^2+b}{2x+c}$.



(a) Determine the value of a, b and c. [3]

$2(1)+c=0$

$c=-2$

$(2,0)$

$0=\dfrac{4a+b}{4-2}$

$0=4a+b$

$(0,2)$

$2=\dfrac{b}{c}$

$2=\dfrac{b}{-2}$

$-4=b$, $a=1$

(b) The function can also be written in the form of $f(x)=px+q+\dfrac{r}{2x+c}$. Determine the values of p, q and r. [3]

$$\begin{array}{r}
\frac{x}{2}+\frac{1}{2} \\
2x-2\,\overline{)\,x^2 \qquad -4} \\
-(x^2-x) \\
\hline
x-4 \\
-(x-1) \\
\hline
-3
\end{array}$$

$f(x)=\frac{x}{2}+\frac{1}{2}-\frac{3}{2x-2}$

$p=\frac{1}{2}$

$q=\frac{1}{2}$

$r=-3$

(c) State the equations of all asymptotes. [2]

$x=1$ $\quad y=\frac{x}{2}+\frac{1}{2}$

7. (6 marks)

Given $f(x) = x^2 - 1$ where x real and $g(x) = \sqrt{9 - x^2}$ where $-3 \le x \le 3$

(a) determine an expression for $f(g(x))$ and state its domain and range [3]

$f \circ g(x) = \left(\sqrt{9 - x^2}\right)^2 - 1$

$D: \quad -3 \le x \le 3 \ , \quad x \in R$

$R: \quad -1 \le y \le 8 \ , \quad y \in R$

(b) determine $h^{-1}(x)$ where $h(x) = f(g(x))$, $-2 \le x \le 0$ [2]

$h^{-1}: \quad x = \left(\sqrt{9 - y^2}\right)^2 - 1$

$x = 9 - y^2 - 1$

$x = 8 - y^2$

$y^2 = 8 - x$

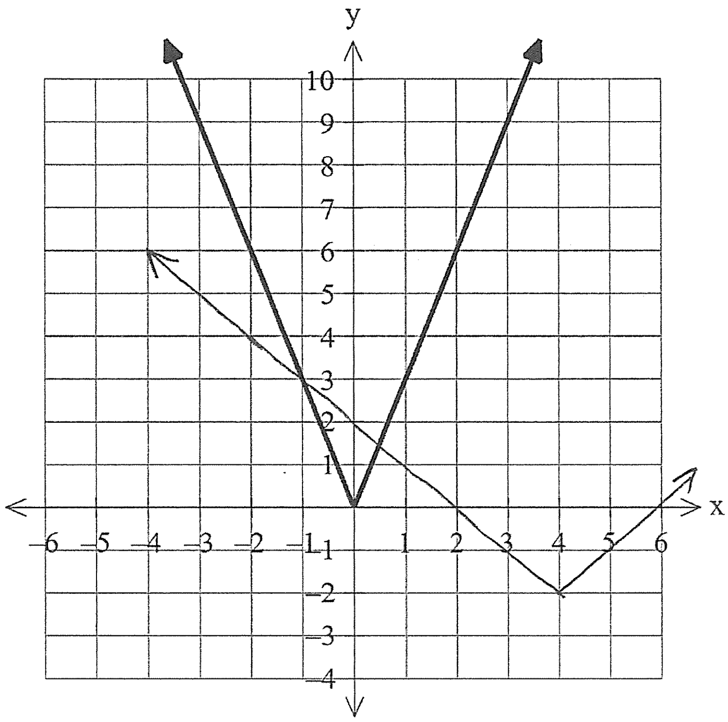
$y = \pm\sqrt{8 - x} \qquad\qquad y = -\sqrt{8 - x}$

(c) state the range of $h^{-1}(x)$ [1]

$R: \quad -2 \le y \le 0 \ , \quad y \in R$

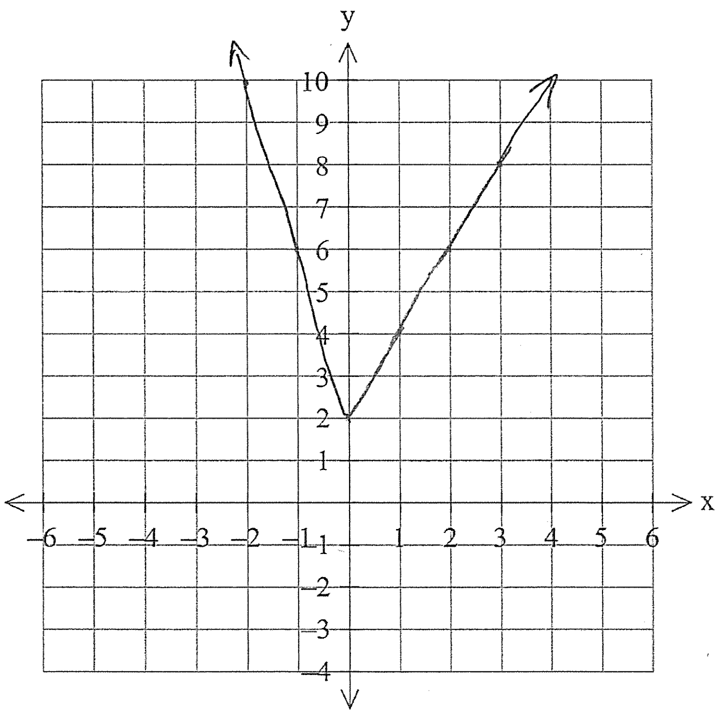
8. (8 marks)

The graph of $y = |3x|$ is drawn on the axes below.



(a) sketch $y = |x-4| - 2$ on the axes above. [2]

(b) sketch $y = |3x| + |x-4| - 2$ on the axes below. [3]



(c) hence solve $|3x| + |x-4| \leq 10$ [3]

$|3x| + |x-4| - 2 = 8$

$\therefore x = 3, -\frac{3}{2}$

$-3 \leq x \leq 8$